The Impact Of PD-LGD Correlation on Expected Loss and Economic Capital

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ABSTRACT

The Basel regulatory credit risk rules for expected losses require banks use downturn loss given default (LGD) estimates because the correlation between the probability of default (PD) and LGD is not captured, even though this has been repeatedly demonstrated by empirical research. A model is examined which captures this correlation using empirically-observed default frequencies and simulated LGD and default data of a loan portfolio. The model is tested under various conditions dictated by input parameters. Having established an estimate of the impact on expected losses, it is speculated that the model be calibrated using banks' own loss data to compensate for the omission of correlation dependence. Because the model relies on observed default frequencies, it could be used to adapt in real time, forcing provisions to be dynamically allocated.

Keywords: Probability Of Default; Loss Given Default; Correlation; Economic Capital; Expected Loss

1. INTRODUCTION

Credit risk affects the vast majority of financial contracts. The measurement, pricing and management of credit risk has received considerable attention from risk managers (to accurately price and manage credit risk) and bank regulators (to assemble and institute capital requirements that correctly identify banks' loan portfolio credit risk). During the recent (2008/9) financial crisis, global financial institutions discovered that the components of credit risk deteriorated in tandem, which led to credit portfolio losses exceeding predictions determined by banks' risk models substantially. Credit portfolio risk is calculated using parameters such as probabilities of default (PD), loss rates given default (LGD), exposures at default (EAD) and dependence variables such as asset and default correlations and loan maturities. These parameters are often modelled independently; dependence structures are added thereafter.

From a regulatory perspective, total credit losses in bank portfolios comprise expected losses and unexpected losses. Unexpected losses lie outside the scope of this research: they are excluded from this investigation because they are affected and influenced by different parameters and, because of multiple conditional dependencies, they require copulae to describe the joint probabilities that define the losses. Copious literature has been produced post credit crisis regarding unexpected losses – the cause of the underestimation in regulatory credit risk capital (e.g. Jorion, 2009; Rohde, 2011), how to correct for this failing (e.g. Avgouleas, 2009; Arewa, 2011), recalibration of parameters influencing the UL (e.g. Bhansali, Gingrich, & Longstaff, 2008; Bassettoa, Cagettid, and de Nardi, 2015), and so on. EL studies have been largely ignored in the literature.

The regulatory approach to EL (and LGD in particular) has been to ignore dependence between PDs and LGDs (BCBS, 2006) and install instead a *downturn* LGD, i.e. one determined during a period of economic stress. As a result, most credit risk models assume LGD is a constant proportion of any credit loss (i.e. that it is a *fixed* value) and ignore the fact that LGD is itself a critical constituent of portfolio credit risk because of its dependence on economic cycles. Using downturn LGD appears to provide the requisite conservativeness to correct for the consequences of ignoring PD/LGD correlation. In adverse economic conditions, GDP diminishes as default frequencies increase and asset prices decrease. As a result, recovery rates decreases (and losses given default *increase*). When economic prosperity resumes, the situation reverses. The capital requirements under these conditions would swing wildly, so to stabilise credit risk capital requirements, through-the-cycle PDs and LGDs are often used.

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Most risk practitioners, however, argue that downturn LGD represents a flawed implementation of the phenomenon that, during an economic crisis, realised loss rates increase in tandem with default rates (see, for example, Pykhtin, 2003; Tasche, 2004; Folpmers, 2012). In addition, because the downturn LGD is calibrated during recessionary periods, it is a biased loss estimate. Consequently, instead of relying on the flawed approach, capital requirements calculated using unbiased LGDs and PD/LGD correlation are becoming more popular (e.g. Rosch & Scheuler, 2014; Eckert, Jakob & Fischer, 2016).

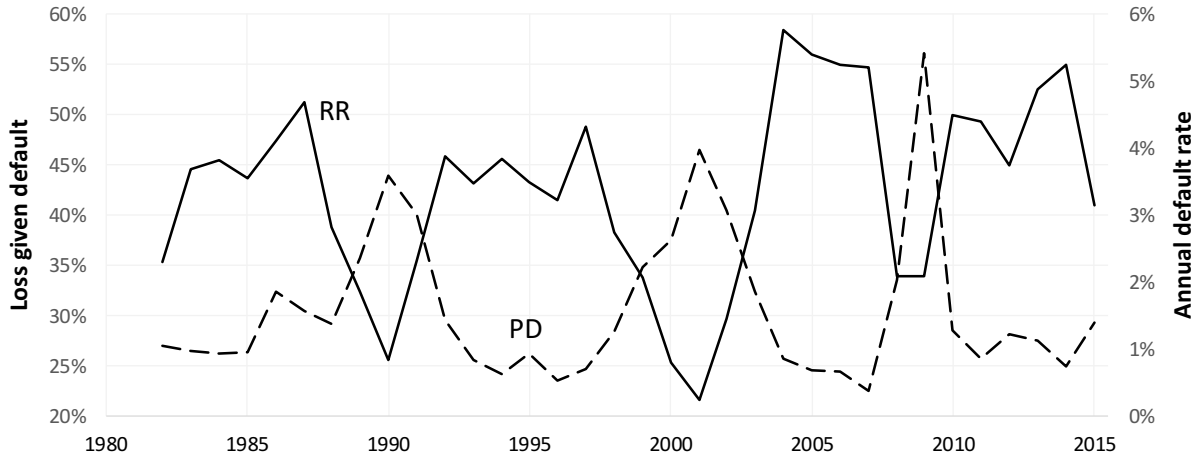
A potential solution lies in modelling increasing loss rates in bad economic states (accompanied by increasing default frequencies) explicitly. This article analyses the impact of the implementation of a model which takes into account observed default frequencies and adjusts the economic capital model accordingly, taking these into account. The model was first proposed by Moody's Analytics (2010) and then adapted by Folpmers (2012) who argued that ignoring this fact risked introducing asymmetry to estimated losses, undermining the normality assumptions and ultimately rending the estimated LGDs inaccurate and invalidating any exogenous calibration. The key input variables in the Folpmers (2012) model are explored to identify their effect on credit portfolio losses. Using empirical data obtained during several financial crises, it is possible to determine broad parameter ranges, outside of which portfolio losses become unrealistic.

The remainder of this paper proceeds as follows: Section 2 provides a literature study which covers the evolution of ideas regarding PD/LGD dependencies. The data used and methodology adopted are given in Section 3, while Section 4 presents and discusses the results. Section 5 concludes.

2. LITERATURE SURVEY

Pykhtin (2003) and Tasche (2004) identified systemic recovery rate volatility as a major source of unexpected credit losses. Other studies argued that default risk (PDs) was linked to economic recoveries (or LGD) (Hu & Perraudin, 2002; Dullmann & Trapp, 2004) as shown in Figure 1.

Figure 1. Global corporate default and recovery rates, 1983-2015.



Source: Moody's Global Corporate Finance (2016).

Ignoring these possibilities could lead to the amplification of economic shocks and could lead to significant underestimation of credit losses. The estimation of PD/LGD correlation is non-trivial, however, and data limitations pose a considerable challenge to banks. Thus, ever since the introduction of the Basel II accord in 2008, regulatory rules have required banks to use *downturn* LGD in their estimation of credit risk capital (BCBS, 2006). The BCBS proposed a principle-based approach in which banks are required to identify relevant downturn conditions and then determine the adverse dependencies between default rates and recovery rates. These must then be incorporated into the bank's credit risk models such that the values identified for the bank's exposures are consistent with observed downturn conditions.

The motivation behind the stipulation of a downturn LGD is that Basel II's Pillar I requirements do not adequately capture the correlations between the probability of default (PD) and the LGD, so the capital requirements for credit risk are underestimated. To compensate for this deficiency, a degree of conservatism should be embedded in the average LGD and an acyclical LGD component should be introduced (see e.g. Pykhtin, 2003, Dullmann & Trapp, 2014; Altman et al. 2005; Giese 2005).

Miu and Ozdemir (2006) found that incorporating a cyclical LGD measure was sufficient to introduce the requisite conservatism to account for the lack of PD/LGD correlation. Two types of correlation were also identified (a PD/LGD correlation between the same borrower and pairwise correlations in LGD risk among a group of borrowers). Results indicated values of 0.53 for the former and 0.06 for the latter amongst large and mid-market corporate borrowers in the 1990s. The mean unbiased estimate of the LGD needed to increase by 37% to reach appropriate levels (i.e. in line with actual losses recorded) of economic capital (Miu & Ozdemir, 2006).

Hillebrand (2006) proposed the use of robust statistical techniques to establish an LGD model that had a 'reasonable economic interpretation' and that could be calibrated using market data. Hillebrand's (2006) model assumes that the LGD is linearly dependent upon several standard normally distributed systemic market factors (Altman et al. 2002) proposed such a model dependent on macroeconomic factors). A multifactor, latent (unobserved) variable framework was used which requires the estimation of fewer parameters and the use of smaller data sets. Hillebrand's (2007) LGD model provided an excellent fit of corporate loss data and addressed the observation that default rates and expected LGD values required two different factors to provide the appropriate fit.

Kim, Kim and Kim (2007) mapped long term average LGDs to downturn LGDs. The mapping employed was quite general, so distributional assumptions could be applied. A β distribution (which has the desirable property that the domain is [0, 1]) was used: parameters for this distribution were established using moment matching of historical LGD averages and variances of realised loss data. The conditional LGD was found to be 42% greater than the standalone LGD, but it was an increasing function of PD, being 32% for PD $< 5\%$ and 61% for $5\% \leq PD \leq 100\%$.

Keijsers (2012) also demonstrated dependencies between macroeconomic variables and LGD using a state space method. Keijsers (2012) constructed a latent variable from observed default rates and LGDs and asserted that this variable could be interpreted as a macroeconomic variable (and thus, it reflects a 'state of the world'). The Kalman filter model results could also be used to adjust LGD to account for cyclicality.

Frye (2013) derived LGD as a function of PD. This approach does not require the calibration of new parameters and simulations showed that LGD function predictions were more accurate than those derived from regression studies. This approach was proposed as a means of reducing forecast noise and it takes PD/LGD correlations into account intrinsically (Frye, 2013).

A joint estimation approach for forecasting PDs and LGDs was proposed by Rosch & Scheuler (2014). This work involved modelling dynamic, consistent and unbiased credit portfolio risks. Measures of economic capital using unconditional parameters were underestimated by up to 17% compared with measured using conditional parameter values.

Eckert, Jakob & Fischer (2016) asserted that empirical evidence exists for dependence between recovery rates, PDs and EADs. A credit portfolio framework, which allowed for the dependencies between these variables – and allowed for the segmentation of LGDs into secured and unsecured recovery rates – was assembled. Using simulated data, Eckert, Jakob and Fischer (2016) found that the portfolio economic capital increased by 31% under dependence between systematic factors compared with independent risk parameters.

Folpmers (2012) argued that the idea of a downturn LGD represented a flawed approach because it was calibrated during economic recessions. Economic reasoning suggests that PD/LGD is dependent upon the economic cycle: downturns caused by a decrease in consumption result in increased PDs, a decrease in the market value of collateral and an associated increase in LGDs. The combination of increased PDs and LGDs leads to the inevitable increase in regulatory credit risk capital. In times of economic prosperity, the above situation is reverse.

To stabilise the capital requirement, Folpmers (2012) asserted that using through the cycle LGDs may be preferable to static LGDs scaled by a factor to account for 'downturns'. Through the cycle LGDs, however, do not take tail risk into account and are thus significantly influenced by the co-movement of observed default frequencies and realised loss rates. The solution proposed was to model LGD increases with increasing PDs in severe economic conditions explicitly (Folpmers, 2012).

In the remainder of this article, Folpmers' (2012) work is used to explore further consequences of the approach on simulated portfolios. The results are compared with observations made during the credit crisis of 2008/9 and in times of economic prosperity.

3. METHODOLOGY AND DATA

3.1 Methodology

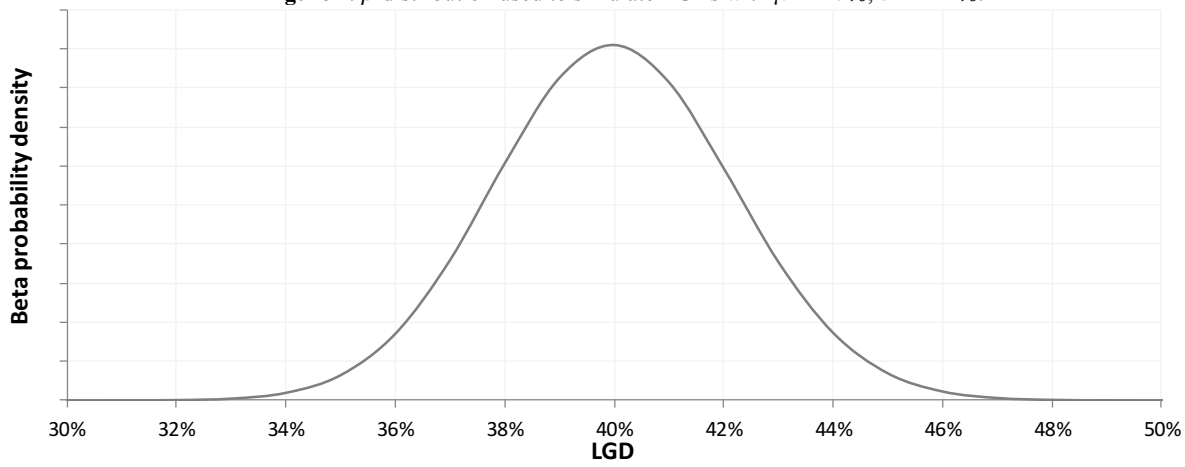
Folpmers (2012) adapted work originally suggested by Moody's Analytics (2010). This framework is then extended and applied to an example portfolio to demonstrate that PD/LGD correlations affect not only EC but also expected loss (EL).

Moody's model (Moody's, 2010) uses a Merton model to determine an obligor's asset returns (r_A):

$$r_A = R_A \cdot \varphi_A + \sqrt{1 - R_A^2} \cdot \varepsilon_A \quad (1)$$

where R_A is the factor loading for the systematic risk, φ_A is the normally distributed systematic risk driver ($\varphi_A \sim N(0,1)$) and the random variable, ε_A, is the driver for the idiosyncratic risk (also normally distributed $\varepsilon_A \sim N(0,1)$). The systemic ($\varphi_A$) and idioscyncratic ($\varepsilon_A$) risks are independent. Within each scenario, for each defaulted loan, a loss rate is determined based on the loan's LGD (simulated from a β distribution with expected value $= LGD$ as shown in Figure 2).

Figure 2. β distribution used to simulate LGDs with $\mu = 40\%$, $\sigma = 2.2\%$.



The loan's recovery rate (r_{RR}) is interpreted as a shock applied to the realised loss rate (LR, drawn from the β distribution). An adjusted loss rate is calculated using:

$$LR_{Adj} = LR \cdot (1 - \Upsilon_{r_{RR}} \cdot r_{RR}) \quad (2)$$

where $\Upsilon_{r_{RR}}$ is a scaling factor applied to the loss rate. Note that simulations of LR_{Adj} must be within their logical $[0, 1]$ domain.

Recovery rate shocks (r_{RR}) are determined using:

$$r_{RR} = R_{RR} \cdot \varphi_{RR} + \sqrt{1 - R_{RR}^2} \cdot \left(\rho_\varepsilon \cdot \varepsilon_A + \sqrt{1 - \rho_\varepsilon^2} \cdot \varepsilon_{RR}\right) \quad (3)$$

where R_{RR} is the factor loading for the recovery rate's systematic risk, φ_{RR} is the driver for the recovery rate's systematic risk and is normally distributed ($\varphi_{RR} \sim N(0,1)$). Note the similarity with (1). The idiosyncratic component of the recovery rate shock is given by $\rho_\varepsilon \cdot \varepsilon_A + \sqrt{1 - \rho_\varepsilon^2} \cdot \varepsilon_{RR}$ which introduces dependency of this component on the asset returns, thereby linking default rates and loss rates for the credit capital requirement. By definition, ε_{RR} is normally distributed ($\varepsilon_{RR} \sim N(0,1)$) and independent from all other random variables.

The recovery rate shock is driven by:

- the systematic driver of recovery risk, φ_{RR}, which affects all obligors equally for a given scenario and which establishes a mutual dependency between recovery rates across defaulted obligors,
- the idiosyncratic risk of the asset return, ε_A, which institutes dependency between defaults and recovery rates. Moody's (2010) specifies this random variable as obligor-specific.
- the idiosyncratic risk of the recovery rate shock, ε_{RR}, and
- the correlation between the asset return (r_A) and the recovery rate shock (r_{RR}). This is calculated using:

$$\rho_{A,RR} = R_A \cdot R_{RR} \cdot \rho_{\varphi_A,\varphi_{RR}} + \sqrt{1 - R_A^2} \cdot \sqrt{1 - R_{RR}^2} \cdot \rho_\varepsilon \quad (4)$$

From (4), the PD-LGD correlation is an increasing function of the following factors:

- the correlation between the systematic risk drivers for the asset return and recovery rate shock $\left(\rho_{\varphi_A,\varphi_{RR}}\right)$. Since both default probability and loss rates are affected by correlated drivers, the PD-LGD correlation will increase for increasing correlation between their drivers; and
- the factor-loading ρ_ε (used to determine the recovery rate shock) for the asset return's idiosyncratic risk driver.

The exposition thus far was included to re-establish the principles of the original Moody's (2010) model. Folpmers (2012) then proposed an important modification to (3).

For each run of a Monte Carlo simulation, a single realisation of a vector of systematic risk drivers, φ_A is generated[1] and the idiosyncratic driver ε_A, for each loan. Within each simulation:

- an obligor defaults if its asset return $r_A < N^{-1}(PD; 0,1)$. Obligors default, therefore, as often as predicted by their PDs and
- a realised loss rate, LR, is established for each defaulting obligor. The realised loss rate consists of a draw from a β distribution (whose expected value equals the obligor's LGD).

A shock is applied to the realised loss to reflect the PD-LGD correlation. For each defaulted loan, the loss is determined using $LR \cdot EAD$. For each run, the total portfolio loss is the sum of the losses over all defaults. Economic capital is difference between the 99.9th percentile and the expected loss.

Using (4), the practical implementation issue arises that a realisation of the shock r_{RR} is only required for loans that have already defaulted. This portfolio subset is characterised by low values of ε_A.[2] When applying (3), then, realisations of ε_A are no longer drawn from a standard normal distribution, so r_{RR} is no longer normally distributed which means that shocks applied to realised losses are no longer symmetric. Expected realised losses after the

[1] For example, these may be asset returns for combinations of sector and geography.
[2] Low values of ε_A are one of two possible causes for default. The second is a low value of the systematic risk driver, φ_A.

application of the shock, then $\neq LGD$. This is not desirable, since the LGD – as an exogenous data input – has already been calibrated.

Folpmers' proposed change to (3) was to introduce a dependency on the observed default frequency, ODF, to replace ε_A:

$$r_{RR} = R_{RR} \cdot \varphi_{RR} + \sqrt{1 - R_{RR}^2} \cdot \left(\rho_\varepsilon \cdot \left[\frac{\overline{PD} - ODF}{\overline{PD}} \right] + \sqrt{1 - \rho_\varepsilon^2} \cdot \varepsilon_{RR} \right) \quad (5)$$

In (5), ODF is the observed default frequency within each simulation run and $\overline{PD}$ is the portfolio average value $\forall$ PDs. $\left[\frac{\overline{PD}-ODF}{\overline{PD}}\right]$ is then a scaling factor which adjusts to the changing (observed) default frequency. As ODF increases, the squared bracket diminishes until, when $\overline{PD} = ODF$ the quantity is 0 and (5) becomes:

$$r_{RR} = R_{RR} \cdot \varphi_{RR} + \sqrt{1 - R_{RR}^2} \cdot \sqrt{1 - \rho_\varepsilon^2} \cdot \varepsilon_{RR}.$$

For values of $\overline{PD} > ODF$, the quantity $\overline{PD} - ODF > 0$ and overall r_{RR} increases as a result. When $\overline{PD} < ODF$, i.e. when the observed default frequency exceeds the average PD, the quantity $\overline{PD} - ODF < 0$ and overall r_{RR} decreases. In this framework, the PD/LGD correlation is dependent upon good and bad macroeconomic states of the world, not the correlation between single-obligor level risk drivers. This was originally derived by Moody's (2010) who asserted that "overwhelming evidence shows that recovery in a default event is closely related to macroeconomic conditions. Recovery is procyclical: during a recessionary period, recovery tends to be lower than during an expansionary period." The BCBS also refers to the LGD dependency on good and bad credit states, rather than on the PD/LGD correlation at the single-obligor level (BCBS, 2005).

Within each simulation run (using (5)) the complete set of defaulted loans must first be established before the loss rate for each single default may be estimated. This arises because the ODF is an argument in the r_{RR} specification and recovery rates need to take (scenario) PDs into account.

In Folpmers's (2012) amended PD/LGD correlation framework, two types of dependency arise:

- between realised losses within a portfolio due to the systematic driver for recovery risk, and
- between the default risk for a given state of the economy and the realised loss.

The obligor-specific risk (as a driver for both PD/LGD correlation) is ignored and the situation summarised in Table 1.

Table 1. Summary of dependencies modelled in Folpmers' (2012) adjusted framework.

PD-LGD correlation risk driver	Example	Model parameter
Systematic driver for the recovery risk	Asset prices that exhibit co-movement independent of business cycle - e.g., due to lags (collection process for real estate can take up to several years); sector-specific collateral values (e.g., stocks, fixed assets)	φ_{RR}
Correlation between business cycle and realised loss rate	During recession, default risk increases and asset prices such as house prices (mortgage loans), commercial real estate prices (Income Producing Real Estate Loans) and stock collateral (Securities Financing Transactions) decline	ρ_ε
Obligor-specific correlation between PD and LGD	If an SME customer defaults, his enterprise-specific collateral declines in value due to bad management	Not modelled

3.2 Data

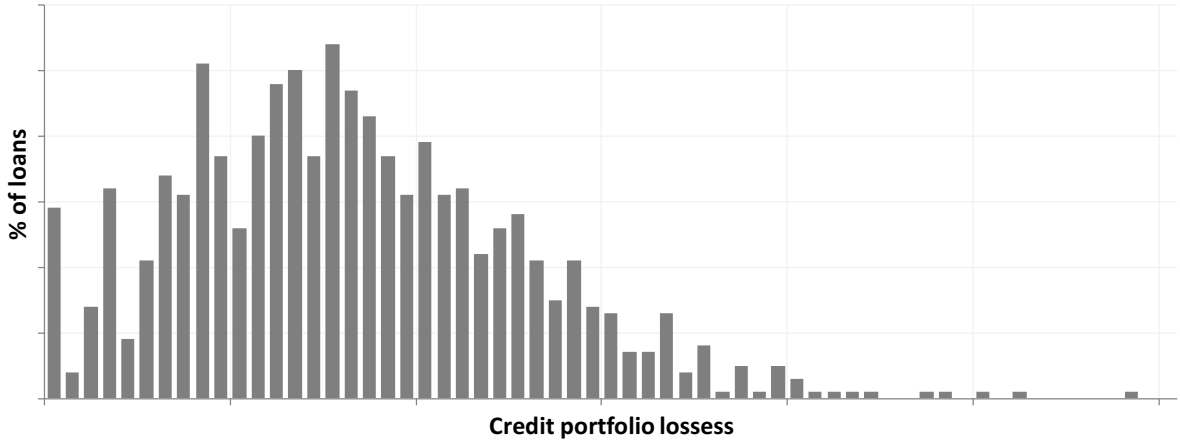
Non-proprietary credit portfolio data are difficult to access and it is more difficult to obtain permission to publish results. As a result, simulated data are generated using Monte Carlo simulations.

Folpmers (2012) proposed PD/LGD correlation model can be demonstrated using an example (simulated) portfolio with the following properties): 10 000 loans; $PDs = 3\%, \overline{LGD} = 40\%$, $EADs = 100$. The economic capital calculation has the following parameters:

- confidence level= 99.9% (in accordance with Basel II regulatory capital for credit risk, BCBS (2006));
- realised loss rates drawn from a β distribution with $a = LGD \cdot n$ and $b = n - a$, $n = 500$ where n is the "level of certainty" about LGDs. A higher value of n means that the loss rates generated using the β distribution are all $\approx \overline{LGD}$, whereas a low value of n means that loss rates are more dispersed across the $[0, 1]$ domain,
- the factor loading for the systematic risk of the asset return R_A is varied
- the elements of the vector φ_A are drawn from a multivariate standard normal distribution using a correlation matrix for which all off-diagonal elements equal 40%. This models a situation in which each pair of sector returns has $\rho = 40\%$,
- 10 000 simulation runs,
- the shock volatility $\Upsilon_{r_{RR}} = 20\%$, (varied for later tests).

Representative credit losses for such a portfolio are shown in Figure 3.

Figure 3. Histogram of characteristic simulated credit portfolio losses, generated using the BCBS (2006) asymptotic single risk factor (ASRF) model.



4. RESULTS AND DISCUSSION

Table 2 presents the inputs used for the base case scenario. For later results, $\overline{(LGD)}$ and n are not altered. Changing the average LGD only changes the average *level* in Figure 3 – 5 and changing n adjusts the spread of simulations. Higher n leads to a "tighter" distribution of simulated values in Figures 3 – 5. Neither of these variables yields any useful insight into the effects of parameter changes on economic capital.

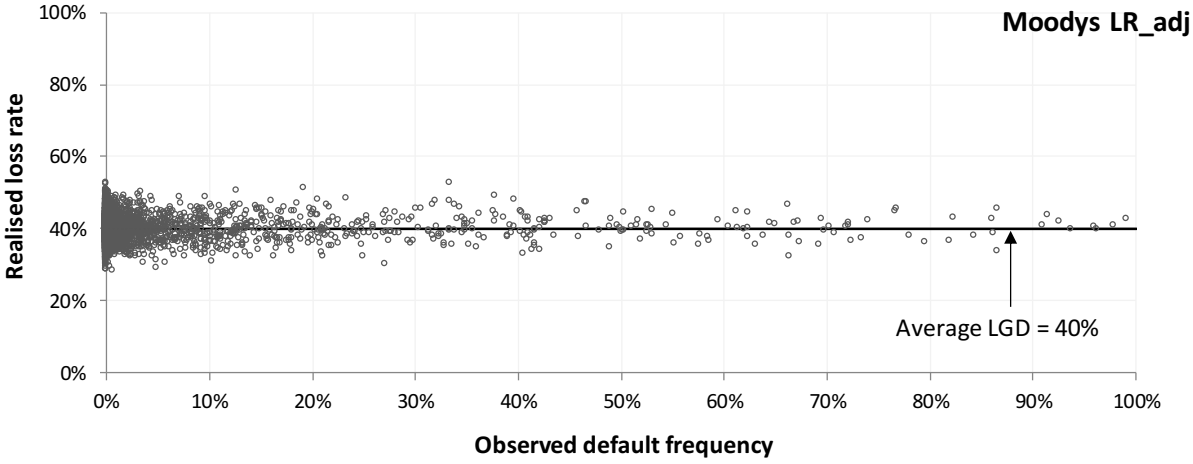
Table 2. Inputs used to produce Figures 3 and 4.

Parameter	Value
$\overline{LGD}$	40%
n	10
R_{RR}	0.2
ρ_{ε}	0.3
$\overline{PD}$	3%
$\Upsilon_{r_{RR}}$	0.2
R_A	0.3

4.1 Base Case

For the base case scenario, PD/LGD correlation $\Upsilon_{r_{RR}} = 0$ — i.e., no shock is applied to the loss rate drawn from the β distribution – see Figure 4. Although these results are obtained for the Moody's LR_{adj} case, the identical results were – as expected – obtained for the Folpmers' LR_{adj} model. The realised loss rate is unaffected by the observed default frequency when the shock is 0.

Figure 4. Moody's adjusted loss rates, using (3) – no shock volatility applied, i.e. $\Upsilon_{r_{RR}} = 0$.



Adjusting the shock factor, $\Upsilon_{r_{RR}}$, has a more dramatic impact on Folpmers (2012) realised loss rate than on Moody's (2010) as seen in Figures 5 and 6 respectively. The Folpmers (2012) approach embeds the observed default frequency explicitly with the result that, increasing the shock factor strongly influences the realised loss rate. The impact increases linearly with increasing observed default frequency. Note that for every observed default frequency the accompanying distribution of realised loss rates is a beta distribution.

Figure 5. Original Moody's adjusted loss rates, using (3) with shock volatility $\Upsilon_{r_{RR}} = 0.2$.

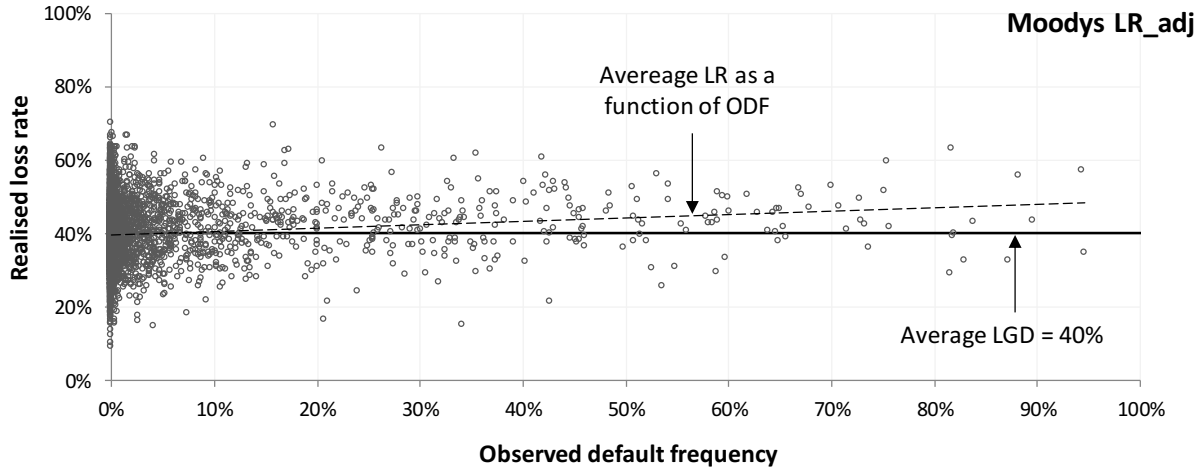
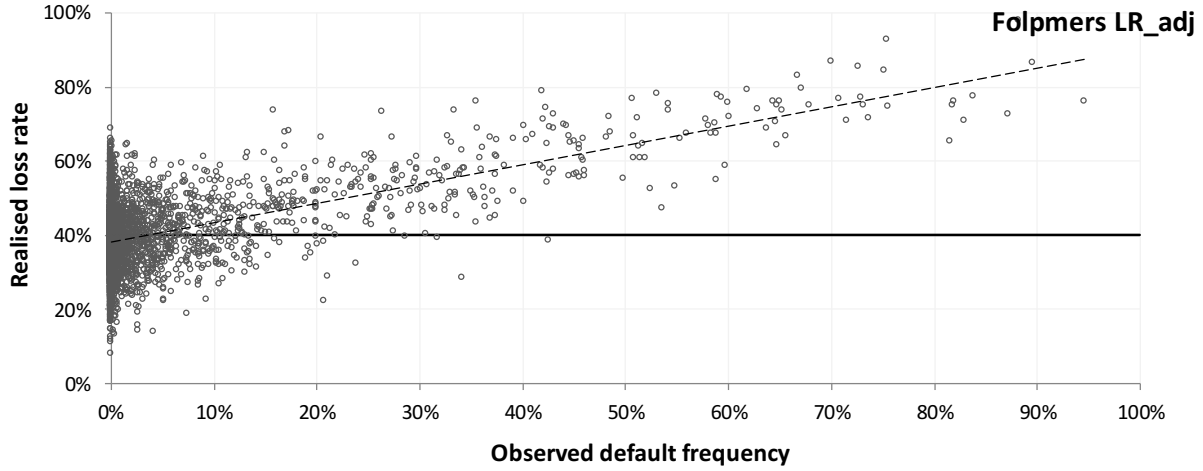


Figure 6. Folpmer's adjusted loss rates, using (5) with shock volatility $\Upsilon_{r_{RR}} = 0.2$.



To establish the relative impact of Folpmers (2012) suggestion, a portfolio with the requisite features (see Section 3) was assembled, the simulations run and four sets of possible portfolio losses examined. The four portfolios were as described in Table 3.

The standard Basel EL was used as the benchmark against which all other ELs were measured, so four relative ELs were obtained – see Table 3.

Table 3. Four EL types calculated from 10 000 loan portfolio simulations.

Loss type	EL calculation ($EAD = 1$)	Relative ELs
Basel EL using fixed (downturn) LGD	$EL = PD \cdot LGD_{fixed}$	Basel EL/Basel EL
Basel EL using LGD sampled from a β distribution	$EL = PD \cdot LGD_{\beta}$	Basel EL$_{\beta}$/Basel EL
Moody's (2010) EL using r_{RR} from (3)	$EL = PD \cdot \left(1 - \Upsilon_{r_{RR}} \cdot r_{RR}\right) \cdot LGD_{\beta}$	Moody's EL/Basel EL
Folpmer's (2012) EL using r_{RR} from (5)		Folpmers' EL/Basel EL

The effect of varying input parameters from (3) and (5) was explored on each of these EL ratios. In each case, the benchmark – the Basel EL (Basel EL/Basel EL) = 1 for all ranges of input parameters. The relative impact of changing

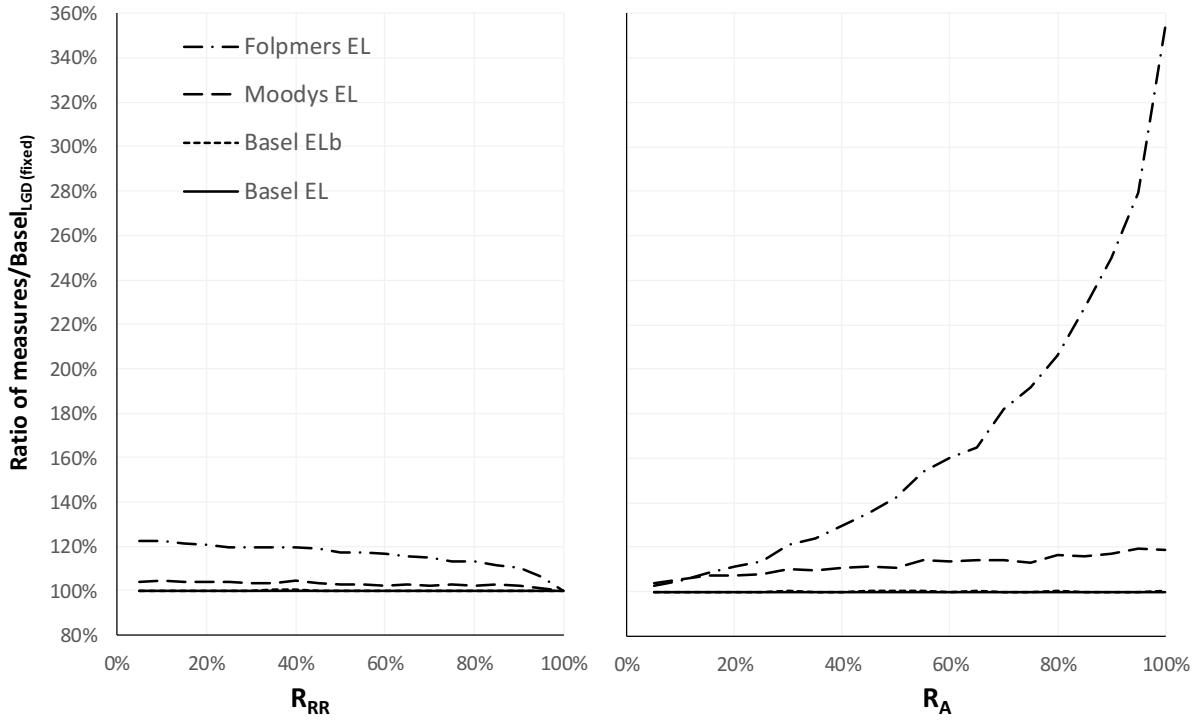
input parameters on the other three ELs is then demonstrated. In each of the figures below (Figure 7 and 8), the same vertical scale is used for comparison.

4.2 Effect of changing R_{RR} and R_A

Recall that R_{RR} is the factor loading for the recovery rate's systematic risk. Low values of this parameter imply that there is low (or no) systematic risk component, all recovery rate risk is idiosyncratic. The opposite is true for high values of R_{RR} – i.e. that the recovery rate comprises only systematic risk. The results shown in Figure 7 (left panel) indicate that the models behave as expected for changes in R_{RR}. Basel's EL_β is unaffected, and Moody's and Folpmer's ELs are higher for low R_{RR}s and the same as the Basel EL for high R_{RR}s. This is because the Basel EL treats the LGD as wholly idiosyncratic by assuming it is a fixed, single value. There is no systematic impact of LGD in the Basel formulation. Moody's and Folpmers take these effects into account, and as the factor loading shifts from the systematic to idiosyncratic component, the impact on EL diminishes and, when the entire recovery rate is assumed idiosyncratic, both Moody's and Folpmer's ELs are identical to Basel's.

The factor loading for the systematic risk, R_A (from (1)) is manifest principally through (4). When R_A is small (or 0), there is no systematic component for asset returns: all risk arises from the idiosyncratic component. This is implicitly modelled in the Basel formulation, which uses Merton's approach in the asymptotic single risk factor model (BCBS, 2006; Kim, Kim & Kim, 2007) but only for the unexpected loss component of credit losses. As the value of R_A increases, the idiosyncratic component becomes dominant – leaving Basel ELs unchanged but increasing both Moody's and Folpmers' EL (in the latter case, depending on parameter choices, by up to a factor of ≈ 3.5). This large difference from the Basel ELs has implications for bank's provisions and pricing and is discussed in the next section.

Figure 7. Ratio of normalised ELs for varying values of factor loading for recovery rate systematic risk R_{RR} (left panel) and factor loading for systematic risk of asset return R_A (right panel).



4.3 Effect of Changing ρ_ε and $\Upsilon_{r_{RR}}$

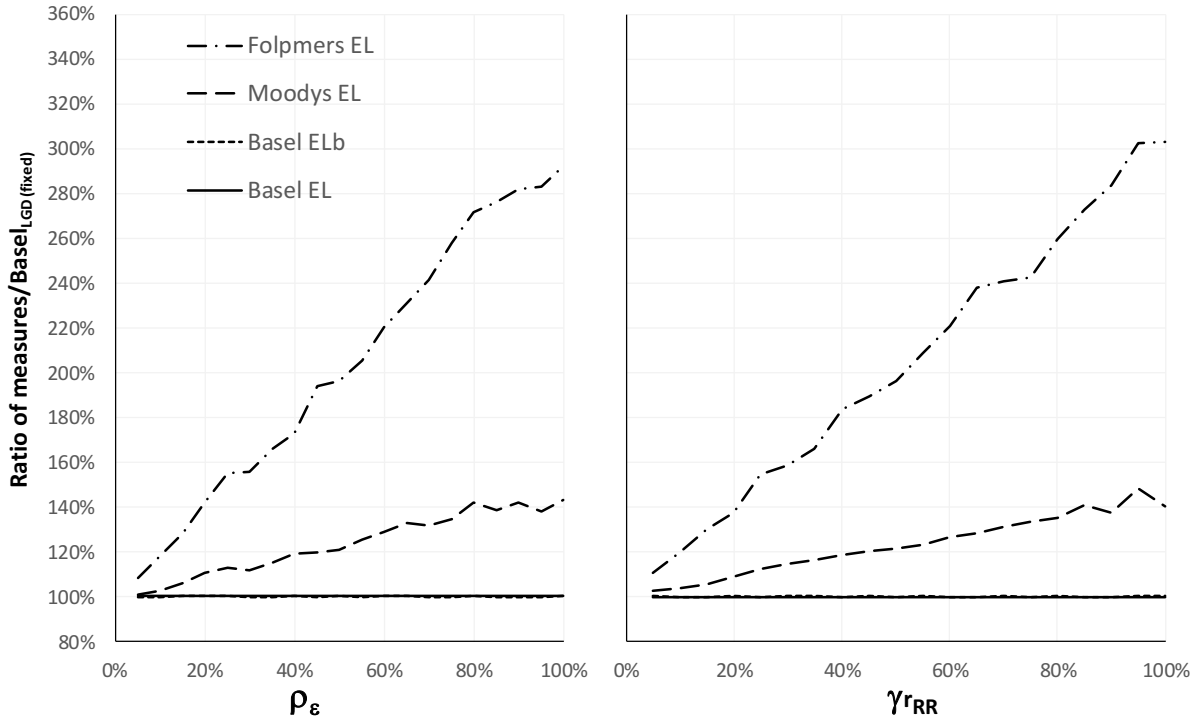
The factor-loading ρ_ε for the asset return's idiosyncratic risk driver, influences the recovery rate shock through (3). Note that the idiosyncratic component of the recovery rate shock is

$$\rho_\varepsilon \cdot \varepsilon_A + \sqrt{1-\rho_\varepsilon^2} \cdot \varepsilon_{RR}$$

(the terms have been defined previously). This component, then, is partially dependent on the asset return, which establishes a dependency between default and loss rates for the credit capital requirement. For low values of ρ_ε, the impact on EL is negligible, but as this value increases, so does the impact on EL for both Moody's and Folpmers. In both cases, the change is roughly linear, up to ≈ 1.5 and ≈ 3, respectively (see left panel of Figure 8). These changers are dependent upon other input parameters, but it is clear that this is a significant variable in the formulation of EL.

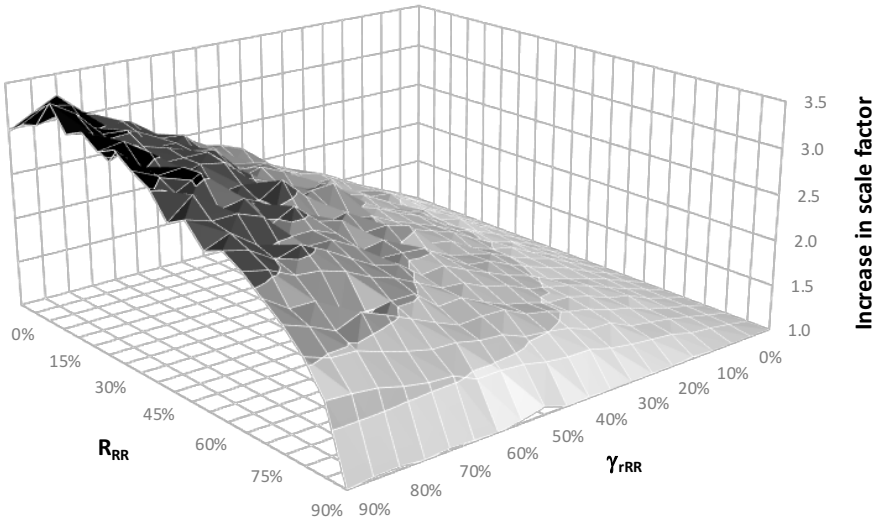
Finally, the shock scaling factor applied to the loss rate, $\Upsilon_{r_{RR}}$, influences ELs in the way shown in the right hand panel of Figure 8. Again, the impact is roughly linear and similar in magnitude to that observed for changes in ρ_ε (and again, dependent upon values chosen for other relevant parameters). The size of the shock factor clearly plays a significant role in the value of EL.

Figure 8. Ratio of normalised ELs for varying values of the factor-loading ρ_ε for the asset return's idiosyncratic risk driver (left panel) and the shock scaling factor applied to the loss rate, $\Upsilon_{r_{RR}}$ (right panel). Note that all vertical scales are the same for comparison.



The effect of the combination of two important variables (the factor loading for the recovery rate systematic risk, R_{RR} and the scaling factor applied to the loss rate, $\Upsilon_{r_{RR}}$) on the normalised Folpmers (2012) EL is shown in Figure 9. The combination of effects – which also depend on other input values – peaks at $\approx 3.5\times$ the original, unscaled Basel EL.

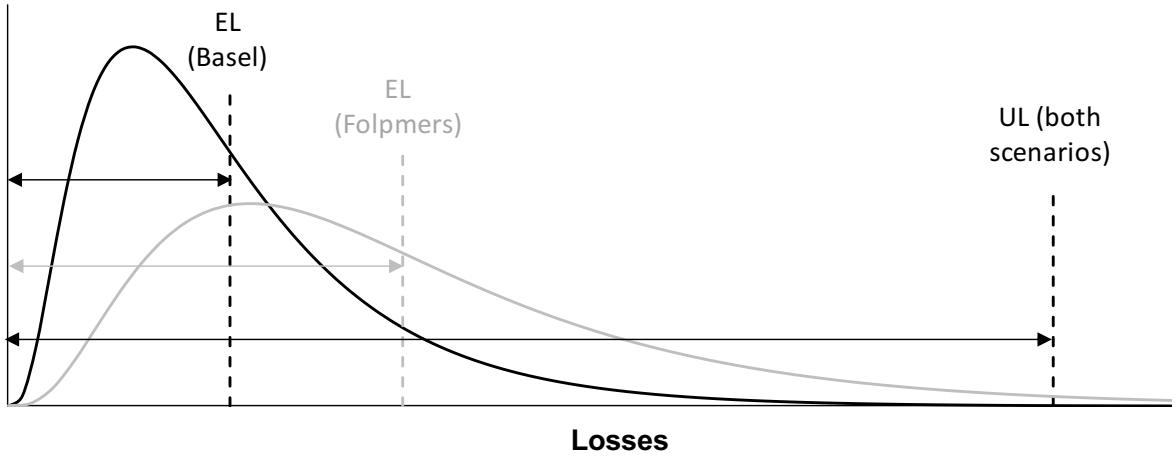
Figure 9. Impact on normalised Folpmers (2012) EL of factor loading for recovery rate systematic risk, R_{RR} and scaling factor applied to the loss rate, $\Upsilon_{r_{RR}}$.



The recent (2008) credit crisis provides an opportunity to calibrate Folpmers (2012) model. In the period leading up to the crisis expected losses greatly exceeded provisions provided by banks triggering demands for enhanced monitoring of provisions and an investigation into alternative models for expected losses (Financial Reporting Council, 2013; Harris, Khan & Nissim, 2013).

Median losses were underestimated by a factor of between 2 to 6 from the actual provisions held by banks (Edmonds, Jarrett & Woodhouse, 2010; Ong, 2014) – or in different terms, the Basel EL estimated at the time should have been between 2 to 6 times larger to provide suitable protection from EL. This is well within the increase in scale suggested by Folpmers' (2012) model. With more data – real not simulated – individual banks could plausibly calibrate the parameters that constitute the Folpmers (2012) model to provide a more accurate, reliable measure of their EL. Since EL depends on observed default frequencies, it could, in principle, be updated and recalibrated as new loss data are recorded – and the EL (and hence provisions for the EL) updated accordingly. In a sense, this could provide a measure of countercyclicality, not unlike the capital buffers proffered by the BCBS to protect banks from the problem of procyclicality (BCBS, 2010) – as demonstrated in Figure 10.

Figure 10. EL and UL using the Basel formulation and Folpmers proposal.



As observed default frequencies increase, the EL would automatically adjust to the new loss environment: provisions could be consequently adjusted. It is plausible that the Folpmers (2012) formulation could increase EL without necessarily increasing the UL. This would mean regulatory capital could, in principle, remain the same even in a deteriorating credit environment, while provisions would constantly be adjusted, swelling to absorb losses when required and diminishing (and thus releasing capital) in benign periods.

5. CONCLUSIONS

Shortly after Basel II's introduction in 2008, criticisms began to emerge about the lack of PD/LGD correlation in the adaptation and formulation of Vasicek's ASRF model. Practitioners complained that the omission penalised them during favourable economic times and academics argued that the use of downturn LGD neither compensated for the cyclicality of the variables, nor addressed the magnitude of the dependency. To date (August 2016) the credit-risk regulatory methodology for taking into account the correlation between these factors remains in place in the current version of the accords. The reason is because of the inherent complexity involved in the mathematical modelling of the relationship and practical obstructions (involving, amongst others, a scarcity of reliable data).

Several methodologies have been introduced since 2008 to address the issue – each comes with associated pros and cons: some are easily implemented, but fare no better than Basel's downturn LGD approach. Others are reasonably accurate and incorporate cyclicality and other known features of the relationship, but are intractable and complex. To date, no universally-accepted solution has emerged.

Moody's (2010) attempt to model correlations between PD and LGD showed promise, but were quickly superseded by Folpmers (2012) more robust approach which struck upon the idea of using *empirical* data in the PD/LGD correlation formulation. Folpmers (2012) idea was to scale the correlation according to observed default frequencies. As these change, the level of dependency adapts.

This article explored the dependence of the parameters required as inputs into Folpmers' (2012) model on one another and, since Folpmers (2012) made no suggestions for the effective calibration of these parameters, the 2008/9 credit crisis (and others), were used as a convenient benchmark against which to test known results with theoretical predictions. The calibration was conducted against the backdrop of expected portfolio losses. Since these are now known, historically, in some detail, the effect of the Folpmers (2012) model (and the magnitude of the input parameters) on EL can be ascertained. The calibration values obtained in this article give approximate values and ranges to be expected during both economically favourable and unfavourable conditions. However, banks will ultimately have to undertake this themselves, depending on their own in-house loss experiences. The process is not onerous, however.

Suggestion for future work could involve explicit calibration for specific bank loss data, suitably anonymised. In addition, results could be compared for expected losses obtained during both upturn and downturn economic conditions to establish cyclicality (or lack thereof) in the results.

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